Math 2412 Instructor: Adnan Said Chapter 4 Handout

4.1 Angles ; Radian and Degree Measure

Angles: An angle is in standard position if its vertex is at the origin of a rectangular coordinate system and its initial side lies along the positive x–axis. Positive angles are generated by counterclockwise rotation, and negative angles by clockwise rotation. An angle θ(theta) lies in a quadrant if the terminal side of the angle lies in that quadrant.

Degree Measure: A measure of one degree is equivalent to a rotation of 1/360 of a complete revolution about the vertex.

Radian Measure: One radian is the measure of a central angle θ that intercepts an arc *s* equal in length to the radius r of the circle.

Conversion from degrees to radians or radians to degrees is based on
360 degrees = 2π radians or 180 degrees = π radians

To convert degrees to radians , multiply by $\dfrac{\pi \text{ rad}}{180^\circ}$.

To convert radians to degrees, multiply by $\dfrac{180^\circ}{\pi \text{ rad}}$.

Convert the angle to radians. Leave as a multiple of π.

1) **240°** $240^\circ \cdot \dfrac{\pi \text{ rad}}{180^\circ} = \dfrac{4\pi}{3}$ radians 1) ___________

Convert the degree measure to radian measure.
Use the value of π found on a calculator and round answers to two decimal places.

2) **– 23.6°** $-23.6^\circ \cdot \dfrac{\pi \text{ rad}}{180^\circ} = -0.41$ radians 2) ___________

Convert the radian measure to degree measure.

3) $\mathbf{\dfrac{17\pi}{12}}$ $\dfrac{17\pi}{12} \cdot \dfrac{180^\circ}{\pi \text{ rad}} = 255^\circ$ 3) ___________

Convert the radian measure to degree measure.
Use the value of π found on a calculator and round answers to two decimal places.

4) **−1.924** $\quad -1.924 \cdot \dfrac{180^\circ}{\pi \text{ rad}} = -110.24^\circ$ $\qquad$ 4) ___________

Coterminal Angles: Two angles are coterminal if they have the same initial and terminal sides. You can find an angle that is coterminal to a given angle θ by adding or subtracting 360°, or by adding and subtracting 2π.

Find the angle of smallest possible positive measure that is coterminal with the given angle.
Name the quadrant in which angle θ lies in.

5) **θ = 1127°** $\qquad$ 5) ___________

Subtract three multiples of 360°, 1127° − 1080° = 47°

So, θ = 1127° is coterminal with 47°.

Find the smallest positive angle less 2π that is coterminal with the given angle.
Name the quadrant in which the angle θ lies in.

6) $\boldsymbol{\theta = -\dfrac{15\pi}{4}}$ $\qquad$ 6) ___________

Add two multiples of 2π, $-\dfrac{15\pi}{4} + 4\pi = \dfrac{\pi}{4}$

So, $\theta = -\dfrac{15\pi}{4}$ is coterminal with $\dfrac{\pi}{4}$.

Minutes and Seconds: 1 degree = 60 minutes, 1 minute = 60 seconds, so 1 degree = 3600 seconds.

To change from minutes to degrees divide by 60.

To change from seconds to degrees divide by 3600.

Convert the angle to decimal degrees and round to the nearest hundredth of a degree.

7) **θ = − 63° 33' 41''** $\qquad$ 7) ___________

$$-63^\circ\, 33'\, 41'' = -\left(63 + \frac{33}{60} + \frac{41}{3600}\right)^\circ \approx -63.56^\circ.$$

Perform the calculation. Express the answer in degree−minutes−seconds format.

8) 180°− 5° 48' 15'' $\qquad$ 8) ___________

180°− 5° 48' 15'' = 179° 59' 60'' − 5° 48' 15'' = 174° 11' 45'' .

Arc–Length Formula: The length s of an arc intercepted by a central angle θ in radians on a circle of radius r is given by **$s = \theta \cdot r$.**

Solve the problem. Round to the nearest hundredth.

9) On a flywheel with a 65.7 mm radius, how long is an arc subtended by a central angle of 263° ? 9) __________

$$s = \theta \cdot r = \left(263^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}\right) \cdot 65.7 \text{ mm} = 301.58 \text{ mm}.$$

Area of a Sector: The area A of a sector with central angle θ in radians in a circle of radius r is given by **$A = \frac{1}{2} \cdot r^2 \cdot \theta$.**

Solve the problem. Round to the nearest hundredth.

10) A sensor light installed on the edge of a home can detect motion for a distance of 68 ft. in front and with a range of motion of 247°. Over what area will the sensor detect motion and become illuminated? 10) __________

$$A = \frac{1}{2} \cdot r^2 \cdot \theta = \frac{1}{2} \cdot (68 \text{ ft})^2 \cdot \left(247^\circ \cdot \frac{\pi \text{ rad}}{180^\circ}\right) = 9966.95 \text{ sq ft.}$$

Angular Speed: If a particle is moving along a circle through an angle of θ radians in time t, then its angular velocity ω(the lowercase Greek letter omega) is given by: **$\omega = \frac{\theta}{t}$.** The angular speed of the particle is the rate at which the angle is changing.

The angular speed does not depend on the radius of the circular path.
In angular speed, 1 complete revolution = 2π rad.

Solve the problem. Round to the nearest hundredth.

11) The blades of a wind turbine are 116 feet long. The propeller rotates at 15 revolutions per minute. What is the angular speed of the propeller in radians per second ? 11) __________

$$\omega = \frac{\theta}{t} = \frac{15 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{30\pi \text{ rad}}{60 \text{ sec}} = \frac{\pi \text{ rad}}{2 \text{ sec}}$$

$= 1.57$ radians per second.

Linear Speed: If a particle is moving along a circle of radius r through an angle of θ radians in time t, then its linear speed v is given by $v = \frac{s}{t}$,

where s is the arc length determined by $s = r \cdot \theta$. So, $\mathbf{v = \frac{s}{t} = \frac{r \cdot \theta}{t} = r\frac{\theta}{t}}$.

The linear velocity of a particle is the rate at which the distance is changing.
In linear velocity, 1 complete revolution = 2π.

Solve the problem. Round to the nearest hundredth.

12) The blades of a wind turbine are 116 feet long. The propeller rotates at 15 revolutions per minute. What is the linear speed of the tips of the blades in inches per second ? 12) ________

$$v = r\frac{\theta}{t} = 116 \text{ ft} \cdot \frac{15 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{12 \text{ in}}{1 \text{ ft}}$$

$= 696\pi$ inches per second = 2186.55 inches per second.

4.1 Exercises pg 505
(17, 27, 31, 39, 57, 63, 73, 75, 98, 113) (19, 25, 33, 37, 61, 69, 74, 76, 99, 114)

4.2 Trigonometric Functions: The Unit Circle

Definitions of Trigonometric Functions

As the real number line is wrapped around the unit circle, each real number t corresponds to a point (x,y) on the circle. That point corresponds to a central angle θ whose radian measure is t. We can use the coordinates of the point to define the six trigonometric functions of t: sine, cosecant, cosine, secant, tangent, and cotangent. These six functions are normally abbreviated sin, csc, cos, sec, tan, and cot, respectively: $\sin t = y$, $\csc t = \frac{1}{y}$, $\cos t = x$, $\sec t = \frac{1}{x}$, $\tan t = \frac{y}{x}$, $\cot t = \frac{x}{y}$

Evaluate the indicated trigonometric functions of each real number.

13) $t = \frac{5\pi}{6}$; sec t, cot t 13) ________

The point on the unit circle that corresponds to t is $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$$\sec\frac{5\pi}{6} = \frac{1}{x} = 1 \div \frac{-\sqrt{3}}{2} = -\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$$

$$\cot\frac{5\pi}{6} = \frac{x}{y} = \frac{-\sqrt{3}}{2} \div \frac{1}{2} = \frac{-\sqrt{3}}{2} \cdot 2 = -\sqrt{3}$$

Domain of Sine and Cosine

Since (x, y) is any point on the unit circle, then $-1 \le x \le 1$ and $-1 \le y \le 1$.
It follows that $-1 \le \cos t \le 1$ and $-1 \le \sin t \le 1$

Period of Sine and Cosine

Adding 2π to each value of t completes a second revolution around the unit circle. The values of $\sin(t + 2\pi n)$ and $\cos(t + 2\pi n)$, where n is an integer, correspond to the values of sin t and cos t.

Even and Odd Trigonometric Functions

The cosine and secant functions are even. $\cos(-t) = \cos t$, $\sec(-t) = \sec t$

The sine, cosecant, tangent, and cotangent functions are odd.
$\sin(-t) = -\sin t$, $\csc(-t) = -\csc t$, $\tan(-t) = -\tan t$, $\cot(-t) = -\cot t$

Use the period to evaluate the sine and cosine.

14) $\sin\left(\frac{13\pi}{6}\right) = \sin\left(\frac{13\pi}{6} - 2\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ 14) __________

15) $\cos\left(-\frac{7\pi}{2}\right) = \cos\left(\frac{7\pi}{2}\right) = \cos\left(\frac{7\pi}{2} - 2\pi\right) = \cos\left(\frac{3\pi}{2}\right) = 0$ 15) __________

Period of Tangent and Cotangent

Adding π to each value of t completes a second revolution around the unit circle. The values of $\tan(t + \pi n)$ and $\cot(t + \pi n)$, where n is an integer, correspond to the values of tan t and cot t.

Use the period to evaluate the tangent and cotangent.

16) $\tan\left(\frac{17\pi}{4}\right) = \tan\left(\frac{17\pi}{4} - 4\pi\right) = \tan\left(\frac{\pi}{4}\right) = \frac{y}{x} = \frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = 1$ 16) __________

17) $\cot\left(-\frac{10\pi}{3}\right) = \cot\left(-\frac{10\pi}{3} + 4\pi\right) = \cot\left(\frac{2\pi}{3}\right) = \frac{x}{y} = -\frac{1}{2} \div \frac{\sqrt{3}}{2}$ 17) __________

$$= -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Reciprocal Identities: $\csc t = \frac{r}{y} = \frac{1}{\sin t}$, $\sec t = \frac{r}{x} = \frac{1}{\cos t}$, $\cot t = \frac{x}{y} = \frac{1}{\tan t}$

Use a calculator to find the value of the trigonometric function to four decimal places.

18) $\cot\left(\frac{11\pi}{6}\right) = -1.7321$ 18) ________

19) $\sec(-37.2°) = 1.2554$ 19) ________

4.2 Exercises pg 520 (11, 21, 47, 54, 67) (13, 23, 51, 59, 69)

4.3 Right Triangle Trigonometry

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle.
The six trigonometric functions of the angle θ are defined as follows:

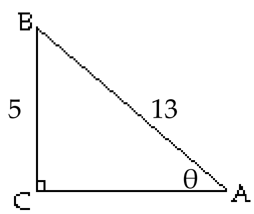
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$, $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$, $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Let θ be an acute angle such that $\sin\theta = \frac{5}{13}$.

Find the values of the remaining trigonometric functions

20) 20) ________



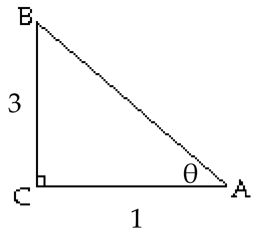
$b^2 = c^2 - a^2 \rightarrow b^2 = 13^2 - 5^2 = 169 - 25 = 144$

$\rightarrow b = \sqrt{144} = 12$

$\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$, $\csc\theta = \frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = \frac{12}{5}$

Let θ be an acute angle such that tan θ = 3.
Find the values of the remaining trigonometric functions.

21) 21) __________



$$c^2 = a^2 + b^2 = 3^2 + 1^2 = 9 + 1 = 10 \rightarrow c = \sqrt{10}$$

$$\sin\theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10},\ \cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10},\ \csc\theta = \frac{\sqrt{10}}{3},$$

$$\sec\theta = \frac{\sqrt{10}}{1} = \sqrt{10},\ \cot\theta = \frac{1}{3}$$

Cofunction Identities

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right),\ \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right),\ \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right),\ \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right),\ \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

Find a cofunction with the same value as the given expression.

22) $\sin\frac{\pi}{9}$ $\quad \sin\frac{\pi}{9} = \cos\left(\frac{\pi}{2} - \frac{\pi}{9}\right) = \cos\left(\frac{7\pi}{18}\right)$ 22) __________

23) csc 37° $\quad \csc 37° = \sec(90° - 37°) = \sec 53°$ 23) __________

Definitions of the Inverse Trigonometric Functions

If $\sin\theta = k$, then $\theta = \sin^{-1}k$

If $\cos\theta = k$, then $\theta = \cos^{-1}k$

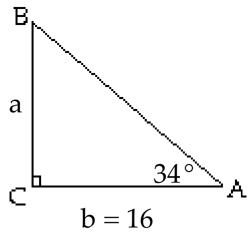
If $\tan\theta = k$, then $\theta = \tan^{-1}k$

Use a calculator to find the acute angle in degrees, rounded to the nearest hundredth.

24) $\sin\theta = 0.4765$ $\quad \theta = \sin^{-1}(0.4765) = 28.46°$ 24) __________

25) $\tan\theta = 7.6538$ $\quad \theta = \tan^{-1}(7.6538) = 82.56°$ 25) __________

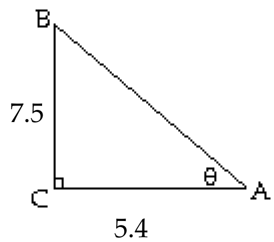
Find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round your answer to the nearest hundredth.

26) 

26) __________

$\tan 34° = \frac{a}{16} \rightarrow \quad a = 16 \cdot \tan 34° = 10.79$

Find the measure of angle θ of the right triangle. Round your answer to the nearest hundredth.

27) 

27) __________

$\tan \theta = \frac{7.5}{5.4} \rightarrow \tan \theta = 1.3889 \rightarrow \theta = \tan^{-1}(1.3889) = 54.25°$

4.3 Exercises pg 533 (3, 23, 31, 37) (5, 27, 33, 39)

4.4 Trigonometric Functions of Any Angle

Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with a point (x,y) on the terminal side of θ, and r is the distance from (0,0) to the point (x,y).

$r = \sqrt{x^2 + y^2} \rightarrow r^2 = x^2 + y^2 \rightarrow x^2 = r^2 - y^2$ or $y^2 = r^2 - x^2$

The six trigonometric functions of the angle θ are defined as follows:

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$, $\csc\theta = \frac{r}{y}$, $\sec\theta = \frac{r}{x}$, $\cot\theta = \frac{x}{y}$

Given that θ is an angle in standard position whose terminal side contains the given point, provide the exact value of the six trigonometric function values.

28) **(2, −1)** 28) __________

$r = \sqrt{x^2 + y^2} = \sqrt{(2)^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}$

$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$, $\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$,

$\tan\theta = \frac{y}{x} = \frac{-1}{2}$, $\cot\theta = \frac{x}{y} = \frac{2}{-1} = -2$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{-1} = -\sqrt{5}$, $\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{2}$

Solve the problem.

29) Find sin θ given $\cos\theta = -\frac{8}{9}$, $\tan\theta > 0$. 29) __________

$\cos\theta = \frac{x}{r} = \frac{-8}{9} \rightarrow x = -8$ and $r = 9$

$y^2 = 9^2 - (-8)^2 \rightarrow + y^2 = 81 - 64 \rightarrow y^2 = 17 \rightarrow y = \pm\sqrt{17}$

Since $\cos\theta < 0$ and $\tan\theta > 0$, then θ lies in quadrant III.

$\rightarrow y = -\sqrt{17}$ So, $\sin\theta = \frac{y}{r} = \frac{-\sqrt{17}}{9}$

30) Find $\cos\theta$ given $\tan\theta = -\frac{8}{3}$, θ in quadrant II. 30) __________

$\tan\theta = \frac{y}{x} = -\frac{8}{3} = \frac{8}{-3} \rightarrow y = 8$ and $x = -3$

$r = \sqrt{(-3)^2 + (8)^2} = \sqrt{9 + 64} = \sqrt{73}$

So, $\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{73}} = \frac{-3\sqrt{73}}{73}$

Reference Angles:

The reference angle for any angle θ is the positive acute angle θ'; read (theta prime) formed by the terminal side of θ and the positive or negative x-axis.

1) If θ lies in quadrant I, then $\theta' = \theta$.
2) If θ lies in quadrant II, then $\theta' = \pi - \theta$.
3) If θ lies in quadrant III, then $\theta' = \theta - \pi$.
4) If θ lies in quadrant IV, then $\theta' = 2\pi - \theta$.

Find the reference angle for the given angle.

31) $\theta = \frac{11\pi}{4}$ $\frac{11\pi}{4} - 2\pi = \frac{3\pi}{4}$ is coterminal with $\frac{11\pi}{4}$. 31) __________

Since $\frac{3\pi}{4}$ lies in quadrant II, then $\theta' = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

32) $\sin(-150°)$ $-150° + 360° = 210°$ is coterminal with $-150°$. 32) __________

Since $210°$ lies in quadrant III, then $\theta' = 210° - 180° = 30°$.

Evaluating Trigonometric Functions Using Reference Angles

$\sin\theta = \pm\sin\theta'$, $\cos\theta = \pm\cos\theta'$, $\tan\theta = \pm\tan\theta'$

$\csc\theta = \pm\csc\theta'$, $\sec\theta = \pm\sec\theta'$, $\cot\theta = \pm\cot\theta'$

The sign is determined by the quadrant in which θ lies.

Use the reference angle to find the exact value of the expression.

33) $\cos\left(\frac{11\pi}{4}\right)$ $\cos\left(\frac{11\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ 33) __________

34) $\sin\left(-150°\right)$ $\sin\left(-150°\right) = -\sin 30° = -\frac{1}{2}$ 34) __________

4.4 Exercises pg 548 (5, 25, 29, 67, 77, 83) (7, 27, 33, 69, 81, 85)

4.5 Graphs of the Sine and Cosine Functions

To graph $y = A \sin(Bx - C)$ or $y = A \cos(Bx - C)$

1. Find the amplitude: Amplitude $= |A|$. The amplitude represents half the distance between the maximum and minimum values of the function.
2. Find the period: Period $= \frac{2\pi}{B}$.
3. Find the key points of the one cycle interval.

   The first point of a one−cycle interval can be determined by $x = \frac{C}{B}$.

   The first point is called the phase shift.
   The second point can be determind by adding quarter period to the first point.
   The third point can be determind by adding quarter period to the second point.
   The fourth point can be determind by adding quarter period to the third point.
   The fifth point can be determind by adding quarter period to the fourth point.
4. Sketch one cycle through the five key points, and one additional cycle (if needed).

Let $y = 2 \sin(2x - \pi)$,

35) **Find the amplitude, period, and quarter period.** 35) __________

Amplitude $= |A| = |2| = 2$ Period $= \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

Quarter Period $= \frac{\pi}{4}$

36) **Find the five key points of the complete cycle.** 36) ___________

The first key point is $x = \frac{C}{B} = \frac{\pi}{2}$; $y = 0$

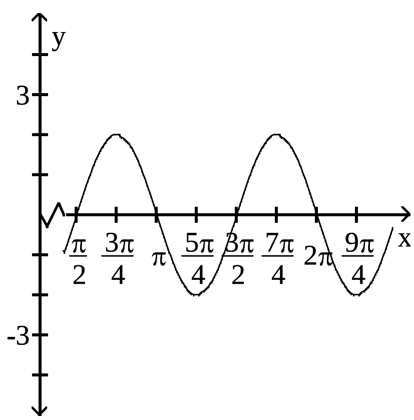
The second key point is $x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$; $y = 2$

The third key point is $x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$; $y = 0$

The fourth key point is $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$; $y = -2$

The fifth key point is $x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$; $y = 0$

37) **Sketch one cycle through the five key points, and one additional cycle (if needed).** 37) ___________



Let $y = 3\cos(4x - \pi)$,

38) **Find the amplitude, period, and quarter period.** 38) ___________

Amplitude $= |A| = |4| = 4$. Period $= \frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$.

Quarter Period $= \frac{\pi}{8}$.

39) **Find the five key points of the complete cycle.** 39) __________

The first key point is $x = \frac{C}{B} = \frac{\pi}{4}$; $y = 3$

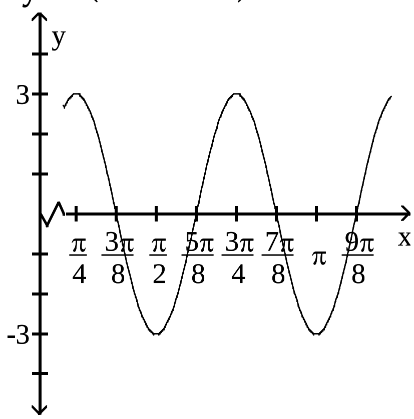
The second key point is $x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$; $y = 0$

The third key point is $x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$; $y = -3$

The fourth key point is $x = \frac{\pi}{2} + \frac{\pi}{8} = \frac{5\pi}{8}$; $y = 0$

The fifth key point is $x = \frac{5\pi}{8} + \frac{\pi}{8} = \frac{3\pi}{4}$; $y = 3$

40) **Sketch one cycle through the five key points, and one additional cycle (if needed).** 40) __________



4.5 Exercises pg 568 (9, 13, 37, 39) (22, 23, 43, 47)

4.6 Graphs of other Trigonometric Functions

To graph $y = A\tan(Bx - C)$

1. The amplitude is not defined.
2. Find the period: Period $= \frac{\pi}{B}$.
3. Two consecutive vertical asymptotes can be determined by solving the equations $Bx - C = -\frac{\pi}{2}$ and $Bx - C = \frac{\pi}{2}$
4. The midpoint between two consecutive vertical asymptotes is an x–intercept of the graph. The distance between two vertical asymptotes is the period of the function.
5. Find the points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes.These points have y–coordinates of $-A$ and A, respectively.
6. Sketch one cycle of the function between the two asymptotes, and one additional cycle (if needed).

Let $y = \tan\left(x + \frac{\pi}{2}\right)$,

41) **Find the amplitude and the period.** 41) __________

Amplitude: Not defined. Period: $\frac{\pi}{B} = \pi$.

42) **Find the equations of the two consecutive vertical asymptotes.** 42) __________

$x + \frac{\pi}{2} = -\frac{\pi}{2} \rightarrow x = -\pi$; $x + \frac{\pi}{2} = \frac{\pi}{2} \rightarrow x = 0$

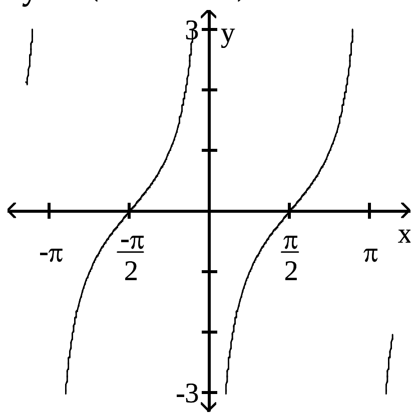
43) **Find the x–intercept of the graph of the function.** 43) __________

The x–intercept is: $\left(\frac{-\pi}{2}, 0\right)$.

44) **Find the points $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes.** 44) __________

The points are: $\left(\frac{-3\pi}{4}, -1\right)$ and $\left(\frac{-\pi}{4}, 1\right)$ respectively.

45) **Sketch one cycle through the five key points, and one additional cycle (if needed).** 45) __________



To graph y = **A cot (Bx – C)**

1. The amplitude is not defined.

2. Find the period: Period = $\frac{\pi}{B}$.

3. Two consecutive vertical asymptotes can be determined by solving the equations Bx – C = 0 and Bx – C = π.
4. The midpoint between two consecutive vertical asymptotes is an x–intercept of the graph. The distance between two vertical asymptotes is the period of the function.
5. Find the points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes. These points have y–coordinates of A and –A, respectively.
6. Sketch one cycle of the function between the two asymptotes, and one additional cycle (if needed).

Let $y = 2 \cot (x + \frac{\pi}{4})$,

46) **Find the amplitude and the period.** 46) __________

Amplitude: Not defined. Period: $\frac{\pi}{B} = \pi$.

47) **Find the equations of the two consecutive vertical asymptotes.** 47) __________

$x + \frac{\pi}{4} = 0 \rightarrow x = -\frac{\pi}{4}$; $x + \frac{\pi}{4} = \pi \rightarrow x = \frac{3\pi}{4}$

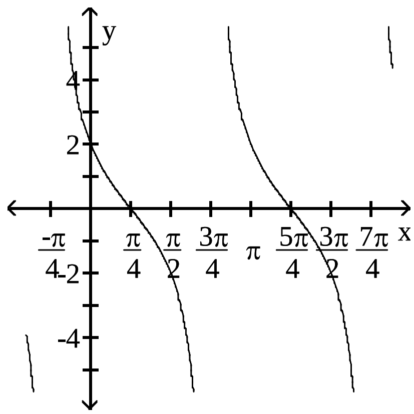
48) **Find the x–intercept of the graph of the function.** 48) __________

The x−intercept is: $\left(\frac{\pi}{4}, 0\right)$.

49) **Find the points $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes.** 49) __________

The point are: $(0, 2)$ and $\left(\frac{\pi}{2}, -2\right)$ respectively.

50) **Sketch one cycle through the five key points, and one additional cycle (if needed).** 50) __________



4.6 Exercises pg 581 (7, 19) (11, 23)

4.7 Inverse Trigonometric Functions

Definitions of the Inverse Trigonometric Functions

If $\theta = \sin^{-1}x$, where $-1 \le x \le 1$, then $\sin\theta = x$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

If $\theta = \cos^{-1}x$, where $-1 \le x \le 1$, then $\cos\theta = x$, where $0 \le \theta \le \pi$

If $\theta = \tan^{-1}x$, where $-\infty < x < \infty$, then $\tan\theta = x$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Find the exact value of the expression.

51) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 51) ________

Let $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \rightarrow \cos\theta = -\frac{\sqrt{2}}{2}$, where $0 \le \theta \le \pi$

The only angle in the interval $0 \le \theta \le \pi$ such that

$\cos\theta = -\frac{\sqrt{2}}{2}$ is $\theta = \frac{3\pi}{4}$.

52) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ 52) ________

Let $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \rightarrow \tan\theta = \frac{\sqrt{3}}{3}$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

The only angle in the interval $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ such that

$\tan\theta = \frac{\sqrt{3}}{3}$ is $\theta = \frac{\pi}{6}$.

Use a calculator to find the value of the expression rounded to two decimal places.

53) $\sin^{-1}\left(-\frac{2}{3}\right) = -0.73$ rad 53) ________

54) $\tan^{-1}\left(\frac{-\sqrt{3}}{5}\right) = -0.33$ rad 54) ________

Composition of Functions Involving Inverse Trigonometric Functions

$\sin(\sin^{-1}x) = x$ where $-1 \le x \le 1$

$\sin^{-1}(\sin x) = x$ where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\cos(\cos^{-1}x) = x$ where $-1 \le x \le 1$

$\cos^{-1}(\cos x) = x$ where $0 < x < \pi$

$\tan(\tan^{-1}x) = x$ where $-\infty < x < \infty$

$\tan^{-1}(\tan x) = x$ where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Find the exact value, if possible.

55) $\cos(\cos^{-1}(0.437)) = 0.437$ 55) ________

56) $\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 56) ________

Let $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) \rightarrow \sin\theta = -\frac{\sqrt{2}}{2}$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

The only angle in the interval $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ such that

$\sin\theta = -\frac{\sqrt{2}}{2}$ is $\theta = -\frac{\pi}{4}$.

57) $\cos(\cos^{-1}(1.5))$ is not defined, since 1.5 is not in $-1 \le x \le 1$. 57) ________

Find the exact value of the composition.

58) $\cos\left(\tan^{-1}\frac{5}{12}\right)$ 58) ________

Let $\theta = \tan^{-1}\left(\frac{5}{12}\right) \rightarrow \tan\theta = \frac{5}{12}$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Since $\tan\theta$ is positive, θ must be an angle in $0 < \theta < \frac{\pi}{2}$.

$r = \sqrt{x^2 + y^2} = \sqrt{(12)^2 + (5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$

$\cos\left(\tan^{-1}\frac{5}{12}\right) = \cos\theta = \frac{x}{r} = \frac{12}{13}$.

59) $\cot\left(\sin^{-1}\frac{1}{3}\right)$ 59) __________

Let $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ $\rightarrow$ $\sin\theta = \frac{1}{3}$, , where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Since $\sin\theta$ is positive, θ must be an angle in $0 < \theta \le \frac{\pi}{2}$

$\sin\theta = \frac{1}{3} = \frac{y}{r}$

$x^2 = r^2 - y^2 \rightarrow x^2 = 3^2 - 1^2 \rightarrow x^2 = 9 - 1$

$\rightarrow x^2 = 8 \rightarrow x = \sqrt{8} = 2\sqrt{2}$

$\cot\left(\sin^{-1}\frac{1}{3}\right) = \cot\theta = \frac{x}{y} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$

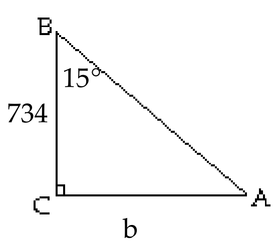
4.7 Exercises pg 547 (5, 21, 35, 47) (17, 25, 41, 57)

4.8 Applications of Trigonometric Functions

Many applications of right triangle trigonometry involve the angle made with an imaginary horizontal line.
The angle formed by a horizontal line and the line of sight to an object that is below the horizontal is called the angle of depression.

From a balloon 734 feet high, the angle of depression to the ranger headquarters is 75°. How far is the headquarters from a point on the ground directly below the balloon (to the nearest foot)?

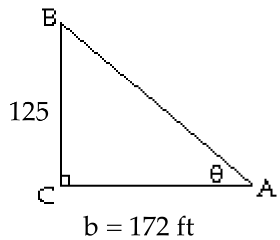
60) 60) __________



$\tan 15° = \frac{b}{734} \rightarrow b = 734 \cdot \tan 15° = 197$ ft.

An angle formed by a horizontal line and the line of sight to an object that is above the horizontal is called the angle of elevation.

A tower that is 125 feet long casts a shadow 172 feet long. Find the angle of elevation of the sun to the nearest degree.

61) 

61) __________

$$\tan \theta = \frac{125}{172} = 0.7267 \rightarrow \theta = \tan^{-1}(0.7267) = 36°.$$

4.8 Exercises pg 609 (3, 34, 41, 45) (9, 35, 43, 46)